***** Sorry...No Solutions are Provided *****

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION
MATA32F - Calculus for Management I

Examiners: R. Grinnell
B.Pike

Date: December 11, 2012
Time: 9:00 am
Duration: 3 hours

Lastname (PRINT): ______________________

Given Name(s) (PRINT): ______________________

Student Number: ______________________

Signature: ______________________

Instructions:

1. This examination has 13 numbered pages. It is your responsibility to check at the beginning of the exam that all of these pages are included.
2. If you need extra space, use the back of a page or blank page 13. Clearly indicate the location of your continuing work. You may write in pencil, pen, or other ink.
3. You may use one standard hand-held calculator that does not perform any of: graphing, matrix operations, numerical or symbolic differentiation or integration. All other electronic devices, extra paper, notes, and textbooks are forbidden at your workspace (visibly, by accident, or intent).
4. If you brought a cell/smart phone into the exam, it must be turned off and left at the front of the exam room.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10	11

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	8	TOTAL
33	14	16	14	20	17	12	12	12	150

Some formulas:

$$S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right] \qquad S = Pe^{rt} \qquad S = P\left(1+\frac{a}{k}\right)^{kt}$$

Part A: Multiple Choice Questions Clearly print the letter of the answer you think is most correct in the boxes on the first page. Right answers earns 3 points and no answer/wrong answers earn 0 points. Justification is not required or rewarded.

1. The value of $\int_0^1 3^x dx$ is (A) $\ln(9)$ (B) $\dfrac{3}{\ln(3)}$ (C) $\dfrac{2}{\ln(3)}$

   (D) $\dfrac{1}{\ln(27)}$ (E) a number not in (A) - (D) (F) undefined

2. If $u = \dfrac{4}{t^2-3}$ then $\dfrac{du}{dt}$ equals

   (A) $\dfrac{tu^2}{2}$ (B) $\dfrac{-t}{u^2}$ (C) $\dfrac{-tu^2}{2}$ (D) $\dfrac{-t}{2u^2}$ (E) none of (A) - (D)

3. What is the greatest amount of compound interest earned at the end of five years if $\$1,000$ is invested today at 2% APR ?

   (A) $\$105.17$ (B) $\$1,105.17$ (C) $\$100$ (D) $\$1,100$

   (E) an amount not in (A) - (D) (F) unknown: we do not know the compounding frequency.

4. $\displaystyle\int_0^3 (24+8t)\,(6t+t^2)^{1/3}\,dt$ equals (A) $\dfrac{27}{8}$ (B) $\dfrac{27}{4}$ (C) $\dfrac{243}{2}$ (D) $\dfrac{243}{8}$ (E) 243

5. The function $y = -16x^4 + 40x^3$ is concave up on the interval(s) (A) $(0, \infty)$
(B) $(1.25, \infty)$ (C) $(-\infty, 0)$ (D) $(0, 1.25)$ (E) both (C) and (B)

6. If y is defined implicitly as a function of x by the equation $xy^2 + \dfrac{x^2}{y} = 6x$ then the slope of the curve at the point $(4, 2)$ is (A) $\dfrac{1}{6}$ (B) $\dfrac{1}{8}$ (C) $\dfrac{-1}{8}$ (D) 2 (E) $\dfrac{-1}{6}$

7. If $\displaystyle\int f(x)\,dx = x^2\ln(x)$ where $x > 0$, then $f'(1)$ equals

(A) 1 (B) 2 (C) 3 (D) 4 (E) a number not in (A) - (D)

8. If $p = mq + b$ is a demand function where $m < 0 < b$, q is quantity, and $0 < q < \dfrac{-b}{m}$ then we have unit elasticity when q equals

(A) $\dfrac{-b}{3m}$ (B) $\dfrac{-b}{2m}$ (C) $\dfrac{b}{2m}$ (D) no value of q (E) a value of q not in (A) - (C)

9. Assume a litre of gas in the Greater Toronto Area increases from \$1.04 to \$1.24 over a 5-month period and that each of the same five percentage price increases occurs only at the end of each month. What is the approximate monthly percentage increase in the gas price?

(A) 3.62% (B) 4% (C) 2.98% (D) 3.58% (E) 0.716% (F) none of (A) - (E)

10. If the average revenue per unit of a product is given by the function $\alpha(q) = 5 + \dfrac{405}{q^2}$ where $q > 0$ is the quantity sold, then the smallest revenue is

(A) \$90 (B) \$5 (C) \$81 (D) \$9 (E) none of (A) - (D)

11. Exactly how many of the following statements are always true?

(i) If $f'(c) = 0$ and $f(c) > 0$ then f has a relative minimum at c.

(ii) If $h(x) > 0$ for all $x > 0$ then $\int_a^b h(x)\,dx$ represents the area under the curve $y = h(x)$.

(iii) The definite integral is a function obtained by taking the limit of a special sum.

(iv) The Extreme Value theorem says that a function defined on a closed interval has both an absolute maximum and minimum on the interval.

(v) Antidifferentiation and differentiation are exactly inverse operations.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5

(Be sure you have printed the letters for your answers in the boxes on the first page)

Part B: Full-Solution Questions Write clear, neat solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32.

1. Find the integrals.

   (a) $\int (e^x + 2x)^2 \, dx$ [7 points]

   (b) $\int \frac{9x}{\sqrt{2+3x}} \, dx$ [7 points]

2. The parts of this question are independent of each other.

(a) If $h(x) = x^a a^{\sqrt{x}}$ where $a > 1$, find $h'(1)$. [4 points]

(b) Briefly justify why the function $f(x) = \dfrac{3\ln(x)}{x}$ where $x \in [2, 5]$ has absolute extrema on the given interval. Find these absolute extrema. [7 points]

(c) Find the equation of the tangent line to the curve $y = e^x$ that passes through the origin. [5 points]

3. The parts of this question are independent of each other.

(a) Imagine you won the recent giant lottery in the U.S.A. and now you are deciding which of Banks A, B, and C to deposit your winnings. Bank A pays 5.4% APR compounding monthly. Bank B pays 5.464% APR compounding semi-annually. Bank C pays 5.39% APR compounding continuously. Determine which bank you should invest your winnings. [7 points]

(b) Find the present value of the generalized annuity where interest is 2.4% APR compounding monthly and $\$3,500$ is deposited at the end of each month for one year, and $\$5,000$ is deposited thereafter at the end of each month for a further 1.5 years. Round your final answer up to the nearest dollar. [7 points]

4. In all of this question let $y = f(x) = 5x^{2/3} - x^{5/3}$

(a) Find all intercepts of f. [2 points]

(b) Find the interval(s) where f is increasing or decreasing, and state all relative extrema. [6 points]

(c) Find the interval(s) of concavity and state the point(s) of inflection. [6 points]

(d) Use all of the information in (a) - (c) to give an accurate sketch of the graph of $y = f(x)$. [6 points]

5. The parts of this question are independent of each other

(a) The demand function for a product is $p = -q^2+100$ and the supply function is $p = \frac{32}{3}q$ where $p \geq 0$ is unit price and $q \geq 0$ is quantity. Find the Consumers' Surplus under market equilibrium, which occurs when $p = 64$. [9 points]

(b) Assume a marginal cost function is $\frac{dc}{dq} = 50 - \frac{20}{q+5}$ where c is the total cost function and $q \geq 0$ is quantity. Assume also that the average cost is $200 per unit when 20 units are sold. Find the fixed cost rounded up to the nearest dollar. [8 points]

6. In all of this question consider a certain positive real number r that has the property:

"the sum of its cube and its square is 20"

(a) State a polynomial function $p(x)$ having r as a root. [2 points]

(b) Find the positive integer a such that r lies between a and b where $b = a + 1$. [3 points]

(c) Let $x_1 = a$ (as in part(b)) and use Newton's Method to find x_2 and x_3.
To three decimal places, what is the approximate value of r? [7 points]

7. In all of this question let $\mathcal{R}$ represent the region that is enclosed between the curves $x = y^2 - 4$ and $x = -(y-1)^2 + 1$.

(a) Give an accurate sketch of $\mathcal{R}$ and show a small rectangle that represents a typical "strip of area". [5 points]

(b) Find the area of $\mathcal{R}$. [7 points]

8. In all of this question consider a piece of wire 10 metres long.

(a) Assume the wire is cut into two pieces. The left piece is bent into a square and the right piece is bent into an equilateral triangle. Where should the cut be made so that the sum of the enclosed areas is a minimum? [10 points]

(The area of an equilateral triangle of side length s is $A = \frac{\sqrt{3}}{4}s^2$)

(b) What can you say about where to cut the wire if the sum of the enclosed areas is a maximum? [2 points]

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