*** Sorry...no solutions are provided ***

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION
MATA32 - Calculus for Management I

Examiners: R. Grinnell
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Date: December 9, 2011
Time: 2:00 pm
Duration: 3 hours

Provide the following information:

Lastname (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number : ______________________________

Signature: ______________________________

Read these instructions:

1. This examination booklet has 13 numbered pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.
2. If you need extra space for any question, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of your continuing work. You may write in pencil, pen, or other ink.
3. You may use one standard hand-held calculator (graphing capability is permitted). All other electronic devices, extra paper, notes, and textbooks are forbidden at your workspace either by accident or intent.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	8	**TOTAL**
30	19	13	16	12	16	10	20	14	150

The following may be helpful:

$$NPV = \left(\sum PV\right) - \text{Initial} \qquad S = Pe^{rt} \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$S = R\left[\frac{(1+r)^{n+1} - 1}{r}\right] - R \qquad A = R + R\left[\frac{1-(1+r)^{-n+1}}{r}\right] \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\text{Profit} = \text{Revenue} - \text{Cost} \qquad \text{Revenue} = (\text{Unit Price}) \times (\text{Quantity}) \qquad \eta = \frac{p/q}{dp/dq}$$

Part A: Multiple Choice Questions For each of the following, **clearly print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 3 points and no answer/wrong answers earn 0 points. No justification is required.

1. If $h'(x) = 3\sqrt{x} + 6x$ and $h(1) = -15$ then $h(4)$ equals

   (A) 48 (B) 44 (C) 36 (D) 34 (E) a number not in (A) - (D)

2. Let c be a positive constant. The area of the enclosed region that lies above the x-axis and below the curve $y = -x^2 + cx$ is

   (A) $\frac{c^3}{6}$ (B) $\frac{c^3}{5}$ (C) $\frac{5c^3}{6}$ (D) $\frac{2c^3}{3}$ (E) not any of (A) - (D)

3. If $y = \sqrt{(x+1)(x+3)(2x+3)}$ then $y'(0)$ equals

   (A) $6\sqrt{3}$ (B) $3\sqrt{6}$ (C) $2\sqrt{6}$ (D) 2 (E) 3 (F) a number not in (A) - (E)

4. If a is a negative constant and $g(x) = axe^{2x}$ then we may conclude that g has

(A) a relative maximum at $x = 1/2$ (C) a relative maximum at $x = -1/2$

(B) a relative minimum at $x = 1/2$ (D) a relative minimum at $x = -1/2$

(E) none of (A) - (D)

5. What amount invested now (rounded up to the nearest dollar) will be worth $\$40,000$ in 5 years if interest is 3% APR compounding quarterly ?

(A) $\$9,147$ (B) $\$13,519$ (C) $\$32,782$ (D) $\$34,448$ (E) $\$35,759$

(F) an amount not in (A) - (E)

6. The least whole number of months it can possibly take for an investment of $\$5,000$ to increase by $\$5,000$ at 2% APR is

(A) 660 (B) 416 (C) 420 (D) 421 (E) a number not in (A) - (D)

(F) unclear because we do not know the frequency at which interest compounds annually.

7. If $a(q) = 2 + \dfrac{128}{q^2}$ is the average cost per unit when $q > 0$ units are manufactured, then the smallest manufacturing cost per unit is

(A) 8 (B) 12 (C) 16 (D) 28 (E) 32 (F) a number not in (A) - (E)

8. The value of $\int_0^1 \left[2+(4x+4)e^{x^2+2x}\right] dx$ is

(A) $2e^3$ (B) $2e^3-2$ (C) $4e^3$ (D) $4e^3+4$ (E) $2+2e^3$

(F) a number not in (A) - (E)

9. If $p=-5q^2+540$ is a demand function where q is quantity and $0<q<10$ then we have unit elasticity when q equals

(A) $4\sqrt{3}$ (B) 4 (C) $4\sqrt{2}$ (D) 6 (E) a number not in (A) - (D)

(F) no number since the elasticity of demand is never unit.

10. Exactly how many of the following statements are always true?

(i) The definite integral is a real number obtained by taking the limit of a special sum.

(ii) Differentiation and antidifferentiation are exactly inverse processes.

(iii) A continuous function h has a local maximum at a number c if and only if c is a critical value of h.

(iv) If $F_1(x)$ and $F_2(x)$ are antiderivatives of a continuous function $f(x)$ then $F_1(x)=F_2(x)$.

(A) 0 (B) 4 (C) 3 (D) 2 (E) 1

(Be sure you have printed the letters for your answers in the boxes on the first page)

Part B: Full-Solution Questions Write clear, full solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32.

1. (a) Find the two points on the curve $y^2 + xy - x^2 = 5$ for which their tangent lines are parallel to the line $x - 2y + 2 = 0$. [11 points]

   (b) Let $f(x) = x^3 - 7$ and let r represent the real root of f. Use Newton's method to find the approximation x_3 to r, where the starting value x_1 is 1 or 2 (depending on whether $f(1)$ or $f(2)$ is closer to 0). Carry five decimals in your calculations and round your final answer to four decimals. [8 points]

2. (a) If you have a choice of investing money at 2.4% compounding daily (365 days $= 1$ year) or 2.41% compounding semi-annually, which is the better choice? [5 points]

(b) Assume interest is 3.6% compounding quarterly. Suppose an initial investment of $\$P$ in a small business will generate the following cash flows (in thousands of dollars):

Year $\rightarrow$	5	10	15
Cash Flow $\rightarrow$	72	83	94

Calculate the largest value of P (rounded down to the nearest \$100) that will guarantee the business investment is profitable. [8 points]

3. In all of this question let the demand function for a certain product be $p = K q^{-1/3}$ where $K > 0$ is a constant, $q > 0$ is the number of units of the product sold, and p is the selling price per unit. Assume each unit of the product costs $m > 0$ dollars to make and every unit made is sold.

(a) Find the domain of the profit function and find the value of q that maximizes the profit. Sufficiently justify your solution.

[11 points]

(b) Show that the point elasticity of demand is a constant and state whether demand is elastic, inelastic, or unit. [5 points]

4. (a) Give a careful statement of the Fundamental Theorem of Calculus. [4 points]

(b) Give a careful statement of the Second-Derivative Test. [4 points]

(c) Use the definition of derivative to find $\lim_{x \to 0} \dfrac{(x+4)^{3/2} + e^x - 9}{x}$

(A solution using l'Hopital's rule (if you know that rule) will earn 0 points). [4 points]

5. Find the exact value of these integrals:

(a) $\displaystyle\int_{1/8}^{1} \frac{6}{x^2}\sqrt{8+\frac{1}{x}}\ dx$ [8 points]

(b) $\displaystyle\int_{1}^{e} \left(ln(x)\right)^2\ dx$ [8 points]

6. In the space below, make axes that extend to ± 7 along the x-axis and from -4 to 5 along the y-axis. Then sketch the graph of a continuous function $y = h(x)$ having **all** of the properties (i) - (v) given below. Points are awarded for accuracy and neatness. Note that there are many quite-different looking graphs for functions that satisfy all of the properties below.

[10 points]

(i) $h(0) = 3$ and $h(-1) = 0$.

(ii) $(5, 2)$ is a point of inflection of h.

(iii) $\lim_{x \to -\infty} h(x) = 0$ and $\lim_{x \to \infty} h(x) = 4$.

(iv) the sign of $h'(x)$ is the same as the sign of $w(x) = \dfrac{x(x+2)(x-3)}{x+1}$ for all $x < 5$ and $x \neq -1$ ("sign" refers to where the function is positive, negative, or zero).

(v) h has no absolute maximum value and has an absolute minimum value of -2.

7. In this question assume $M =$ millions and $km =$ kilometres. An oil drilling rig is located at the point $R = (0, 12)$ offshore and the oil refinery is located onshore at the point $Q = (20, 0)$. Oil is pumped to the refinery through pipe. Underwater pipe costs \$50 M/km and onshore pipe costs \$30 M/km. When either type of pipe is used, it is laid in a straight line segment.

   (a) Let P be the point onshore that lies x km to the left of the point Q where $0 \leq x \leq 20$. Show that the total cost of pipe linking R to P and then P to Q is given by the function $c = 50\sqrt{144 + (20 - x)^2} + 30x$.
   (A labeled diagram and the Pythagorean theorem may be useful).

   [5 points]

   (b) Use (a) to find the least total cost of pipe where $x \in [0, 20]$. State the lengths of the underwater portion of pipe and the onshore pipe. Show all work. [15 points]

8. In all of this question let $\mathcal{R}$ represent the enclosed region in the xy-plane that lies between the curves $x + y^2 + 1 = 0$ and $x - y - 1 = 0$ where $-1 \leq y \leq 1$.

(a) Give an accurate, labeled sketch of the region $\mathcal{R}$. [5 points]

(b) Find the exact area of the region $\mathcal{R}$. Show all work. [9 points]

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