***** Sorry...No Solutions will be provided *****

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION
MATA32 - Calculus for Management I

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Date: December 14, 2010
Duration: 3 hours

Provide the following information:

Lastname (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number : ______________________________

Signature: ______________________________

Read these instructions:

1. This examination has 14 numbered pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.
2. If you need extra space for any question, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of your continuing work. You may write in pencil, pen, or other ink.
3. You may use one standard hand-held calculator (graphing capability is permitted). All other electronic devices, extra paper, notes, and textbooks are forbidden at your workspace.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10	11	12

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	**TOTAL**
48	15	16	15	12	15	10	19	150

The following may be helpful:

$$S = P(1+r)^n \qquad S = Pe^{rt} \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$S = R\left[\frac{(1+r)^{n+1} - 1}{r}\right] - R \qquad A = R + R\left[\frac{1-(1+r)^{-n+1}}{r}\right]$$

$$\text{Profit} = \text{Revenue - Cost} \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \eta = \frac{p/q}{dp/dq}$$

Part A: Multiple Choice Questions For each of the following, **clearly print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 4 points and no answer/wrong answers earn 0 points. No justification is required.

1. If $g(x) = ln(x^2 + 2x)$ then the value of $g'(-1)$ is

   (a) -1 (b) 0 (c) 2 (d) a number not in (a) - (c) (e) undefined

2. If $u'(t) = 6t - 7$ and $u(4) = 35$ then $u(2)$ equals

   (a) -2 (b) 2 (c) -13 (d) 13 (e) 39 (f) a number not in (a) - (e)

3. The value of $\int_0^1 (x+1)\sqrt{x^2 + 2x}\, dx$ is

   (a) $\sqrt{3}$ (b) 3 (c) $2\sqrt{3}$ (d) $\sqrt{3}/2$ (e) a number not in (a) - (d)

4. The equation of the tangent line to the curve $y^2 - y = 2ln(x)$ at the point $(x, 1)$ on the curve is

(a) $y = x$ (b) $y = 3x - 2$ (c) $y = 2x - 1$ (d) $y = -x + 2$

(e) an equation not in (a) - (d) (f) uncertain because we do not know the value of x

5. What is the smallest whole number of days it takes a principal to increase by 15% at 10% APR interest compounding five times per year ? (1 year = 365 days)

(a) 511 (b) 516 (c) 517 (d) 584 (e) a number not in (a) - (d)

(f) uncertain because we do not know the value of the principal

6. Let $\alpha \in (0, 1)$ be a constant and let $c = q^\alpha$ be a cost function where $q > 0$ is quantity. Which of the following statements is true?

(i) the average cost function is increasing (ii) the average cost function is decreasing

(iii) the marginal cost function is increasing (iv) the marginal cost function is decreasing

(a) (i) and (iii) (b) (i) and (iv) (c) (ii) and (iii) (d) (ii) and (iv)

(e) we cannot be sure because the exact value of α is not known

7. If $y = \left(\frac{e}{x^2}\right)^x$ then $y'(1)$ equals

(a) $e/2$ (b) $-e/2$ (c) 1 (d) $-e$ (e) e (f) a number not in (a) - (e)

8. If $\int G(x)\, dx = xe^{-x} + 5$ then $G(1)$ equals

(a) $5+(2/e)$ (b) $2/e$ (c) 0 (d) $5+(1/e)$

(e) uncertain because we cannot find the function $G(x)$ (f) none of (a) - (e)

9. Exactly how many of the following statements are always true?

(i) If a function f is continuous at a number a then f is also differentiable at a

(ii) If n is a real number then the antiderivative of x^n is $\frac{x^{n+1}}{n+1} + C$

(iii) A function h has a relative extremum at a number c if and only if c is a critical value of h

(iv) If $F(x)$ and $G(x)$ are antiderivatives of a function $w(x)$ then $F(x) = G(x)$

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

10. A one-year computer lease consists of two parts: a one-time \$599 insurance fee, plus a weekly rental fee of \$50 which is payable at the end of each of the 52 weeks in the year. The rental fee is subject to interest of 5.2% APR compounding weekly. The insurance fee is not subject to any interest. How much is the total lump-sum computer lease payment if it is paid at the beginning of the year (rounded up to the nearest dollar)? Assume that "one-year" equals 52 weeks.

(a) $\$3,132$ (b) $\$3,265$ (c) $\$3,036$ (d) $\$3,199$ (e) none of (a) - (d)

11. The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 - \frac{k}{n}\right)\frac{1}{n}$ is

(a) $3/2$ (b) 3 (c) $-3/2$ (d) -2 (e) a number not in (a) - (d)
(f) impossible to find

12. The area of the region bounded by the x-axis, the lines $x = -1$ and $y = 2$, and the curve $y = \sqrt{x}$ is

(a) $2/3$ (b) 4 (c) $22/3$ (d) $16/3$ (e) $14/3$

(Be sure you have printed the letters for your answers in the boxes on the first page)

Part B: Full-Solution Questions Write clear, full solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32.

1. In all of this question let $f(x) = e^x - 6x + 3$

   (a) Find the exact values of the absolute extrema of f on $[0, 2]$. Sufficiently justify your solution. [8 points]

   (b) State the Newton method formula. Assume f has a root r in the interval $[0, 1]$. Use Newton's method to find the approximation x_2 to r, where the starting value x_1 is 0 or 1 (depending on whether $f(0)$ or $f(1)$ is closer to 0). Carry five decimals in your calculations and round your final answer to three decimals. [7 points]

2. In all of this question let the demand function for a certain product be $p = \dfrac{D}{\sqrt{q}}$ where $q > 0$ is the number of units of the product sold, p is the selling price per unit, and $D > 0$ is a constant. Assume each unit costs k dollars to make and that every unit made is sold.

(a) Find the value of q that maximizes the profit and find the maximum profit. Sufficiently justify your solution. [10 points]

(b) Show that the point elasticity of demand is a constant and determine whether demand is elastic, inelastic, or unit. [6 points]

3. Find these integrals

(a) $\displaystyle\int \frac{\left(4+\sqrt[3]{x}\right)^{1/5}}{x^{2/3}}\, dx$ [5 points]

(b) $\displaystyle\int_1^4 \sqrt{x}\, ln(x^6)\, dx$ [10 points]

4. In the space below, make axes that extend to ± 7 along the x-axis and from -3 to 7 along the y-axis. Then sketch the graph of a continuous function $y = f(x)$ having **all** of the properties (i) - (v) given below. Points are awarded for accuracy and neatness. Note that there are many quite-different looking graphs for functions that satisfy all of the properties below.

[12 points]

(i) $f(-2) = 3$ and $f(6) = 0$ and $f'(-2)$ is undefined

(ii) $(-4, -2)$ is a point of inflection

(iii) $f''(x) > 0$ for all $x \in (-4, -2)$

(iv) $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f'(x) = 1$

(v) the sign of $f'(x)$ is the same as the sign of $p(x) = x(x-2)^2(x-4)$ for $x \in (-1, 6)$

5. (a) Assume you deposit $\$R$ into ordinary Annuity 1 at the end of every 6 months after your 18^{th} birthday up to and including your 40^{th} birthday (thus you make 44 deposits). Interest is 4% APR compounding semi-annually. After your last deposit at age 40, no further deposits into Annuity 1 are made and its value is then left to accumulate interest at 6% APR compounding semi-annually up to and including your 60^{th} birthday.

At the end of each year starting after your 50^{th} birthday, you now deposit $\$(R+1500)$ into ordinary Annuity 2 up to and including your 60^{th} birthday (thus you make 10 deposits). Interest is 4.8% APR compounding annually.

Find the value of R so that immediately after your 60^{th} birthday, you will have exactly \$1 million dollars in combined total from Annuity 1 and Annuity 2. Round your answer for R up to the nearest dollar. [9 points]

(b) Let $A = R\left[\dfrac{1-(1+r)^{-n}}{r}\right]$ represent the present value of an ordinary annuity considered as a function of the variable n. Verify that $\dfrac{dA}{dn} + A\,ln(1+r) = B\,ln(1+r)$ where $B = R/r$.

[6 points]

6. State the integral(s) that give the total area of the region that is located: on and below the line $y = 2$, and on and to the right of the y-axis, and enclosed between the two curves $x - y = 2$ and $x + y^2 = 4$. You need only give the integral(s); you need not evaluate. Points are awarded for a good sketch. Show all work. [10 points]

7. A research study has determined that sales of a particular *iPod* are expected to grow at a rate of

$$R(t) = 2,000 - 1,500e^{-.05t}$$

units per week where t is the number of weeks after they are introduced into a certain city market. The study assumes that t is a real number ≥ 0, not necessarily a whole number.

(a) [6 points]

(i) Find the expected initial growth rate of sales.

(ii) Calculate the "long-run" growth rate of sales (i.e the growth rate as t becomes large without bound)

(iii) Verify that the rate of sales growth is increasing for all $t \geq 0$

(b) Find the function $N(t)$ that gives the "total number" of *iPods* that are expected to sell throughout the first t weeks after they are introduced into the city market. Assume $N(0) = 0$ and that $N(t)$ need not actually give a whole number. [8 points]

(c) If the retail price of an *iPod* is \$200, find the expected sales revenue for over the first six weeks after they are introduced into the city market. You should not "round" $N(t)$ to a whole number amount, but you should "round" your answer for the expected sales revenue up to the nearest dollar. [5 points]

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