*** Sorry...solutions will not be posted***

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA32 - Calculus for Management I

Examiners: R. Grinnell
G. Pete
B. Szegedy

Date: December 9, 2009
Time: 9:00 am
Duration: 3 hours

Provide the following information:

Surname (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number : ______________________________

Signature: ______________________________

Read these instructions:

1. This examination has 13 numbered pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.
2. If you need extra space for any question, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of your continuing work. You may write in pencil, pen, or other ink.
3. You may use one standard hand-held calculator. All other electronic devices, extra paper, notes, and textbooks are forbidden at your workspace.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	8	**TOTAL**
40	16	15	15	12	12	13	17	10	150

The following may be helpful:

$$S = P(1+r)^n \qquad S = Pe^{rt} \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

Part A: Multiple Choice Questions For each of the following, **clearly print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 4 points and no answer/wrong answers earn 0 points. No justification is required.

1. If y is defined implicitly as a function of x by the equation $xe^y + y = 2$ then y' evaluated at $x = 2$ and $y = 0$ is equal to

   (a) 0 (b) -1 (c) $-1/3$ (d) $1/3$ (e) none of (a) - (d)

2. If $y' = \dfrac{5}{\sqrt{x}}$ and $y(9) = 32$ then $y(4)$ is equal to

   (a) $-94/3$ (b) $59/2$ (c) 27 (d) 22 (e) none of (a) - (d)

3. The area of the region bounded by the curve $y = 3x^2 - 3$, the $y-$axis, $x-$axis, and the line $x = 2$ is equal to

   (a) 9 (b) $13/3$ (c) 2 (d) 6 (e) none of (a) - (d)

4. To what approximate amount will $\$1,200$ accumulate in 20 years if it is invested at an effective rate of 2.4% per year ?

(a) We cannot find the amount because the number of interest periods per year is not given.

(b) $\$1,938.36$ (c) $\$1,928.33$ (d) $\$1,517.35$ (e) none of (a) - (d)

5. The equation of the tangent line to the curve $y = \dfrac{x+5}{x^2}$ at the point where $x = 1$ is

(a) $y = -9x + 15$ (b) $y = 13x - 7$ (c) $y = -12x + 18$ (d) $y = -11x + 17$

6. The exact value of $\displaystyle\int_e^{e^3} \frac{\ln(x)}{x}\,dx$ is

(a) 4 (b) 8 (c) $-4/9$ (d) 1 (e) none of (a) - (d)

7. If $p = -3q + 60$ is a demand function where $0 < q < 20$ then we have unit elasticity at

(a) no value of q (b) $q = 20$ (c) $q = 15$ (d) $q = 10$

(e) a value of q not given in (a) - (d)

8. If $g(x) = xe^{kx}$ where $k < 0$ is a constant, then we may conclude that g has

(a) a relative maximum at $x = -1/k$

(b) a relative minimum at $x = -1/k$

(c) a relative maximum at $x = 1/k$

(d) a relative minimum at $x = 1/k$

(e) none of (a) - (d)

9. If $\int h(x)\, dx = \sqrt{4x+1} + 32$ then $h'(x)$ equals

(a) $2(4x+1)^{-1/2}$

(b) $4(4x+1)^{-3/2}$

(c) $-2(4x+1)^{-3/2}$

(d) $-4(4x+1)^{-3/2}$

10. Exactly how many of the following four mathematical statements are always true:

(i) If a function is differentiable at a then it is also continuous at a.

(ii) If $f''(c) = 0$ then f has a point of inflection at $x = c$.

(iii) The definite integral of a function gives the area under the graph of the function.

(iv) $\int_a^b g(x)\, dx = G(b) - G(a)$ whenever g is continuous and $\int g(x)\, dx = G(x)$.

(a) 4

(b) 3

(c) 2

(d) 1

(e) 0

(Be sure you have printed the letters for your answers in the boxes on the first page)

Part B: Full-Solution Questions Write clear, full solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32.

1. (a) Find the exact value of $y'(1)$ where $y = \dfrac{e^{(x^2+1)}}{\sqrt{x^2+1}}$ [8 points]

   (b) Explain mathematically why the function $f(x) = x^3 - 12x + 1$ has absolute extrema on $[-1, 3]$ and then find these extrema. [8 points]

2. (a) Find $\int x^5 \sqrt{x^3+6}\, dx$ [8 points]

(b) Calculate the exact value of $\int_0^1 4xe^{-2x}\, dx$ [7 points]

3. (a) A debt of $\$4,000$ due in 6 years and $\$6,000$ due in 4 years is to be repaid by a single payment of $\$2,000$ now and three equal payments that are due at the end of each consecutive year, starting from the end of the first year. If the interest rate is 3.2% A.P.R. compounding quarterly, how much are each of the equal payments ? (Round your final answer up to the nearest dollar. A money-time diagram and equation of value are required for full points.) [8 points]

(b) A car is purchased for $\$5,000$ down now and payments of $\$350$ at the end of each month for five years starting from the end of the first month. If interest is 2.4% A.P.R. compounding monthly, find the "cash purchase" of the car. ("cash purchase" = full price, paid now) (Round your final answer up to the nearest dollar.) [7 points]

4. In all of this question, let $p(x) = x^3 + 6x + 1$.

(a) Justify mathematically why p has a root in the interval $[-1, 0]$. [4 points]

(b) Let $x_1 = -0.5$ and use Newton's method to find x_3 as an approximation to the root in part (a). Round your final answer for x_3 to 3 decimals. [8 points]

5. (a) Calculate the maximum amount of compound interest (expressed as a percentage, rounded to 2 decimals) that can be earned over a six year period with 4% A.P.R. [5 points]

(b) Assume the limit $\lim_{x \to -2} \dfrac{x^3 + 5x^2 + cx}{x^3 + 3x^2 + 2x}$ exists. Find the value of the constant c and find the value of the limit. [7 points]

6. In all of this question, assume inventory cost per month is given by the function

$$C(x) = \frac{KD}{x} + aD + \frac{x}{4}$$

where K, D, and a are positive constants and $x > 0$ is the quantity of units stored as inventory per month.

(a) Find the value of x that minimizes the monthly inventory cost. Sufficiently justify your solution.

[8 points]

(b) Find a (in terms of K and D) under the assumption that the minimum monthly inventory cost is $33\sqrt{KD}$

[5 points]

7. In all of this question, assume $y = f(x)$ is a continuous function with the properties:

(i) $f(0) = 3$

(ii) $f(x) > 0$ for all $x \in [-6, 4]$

(iii) $f'(x) = \dfrac{-(x-2)(x+1)}{x^2+1}$

(iv) $f''(x) = \dfrac{-x^2 - 6x + 1}{(x^2+1)^2}$

(a) Find the intervals where f is increasing/decreasing and determine where all relative extrema occur. [6 points]

(b) Find the intervals where f is concave up/concave down and determine where the point(s) of inflection occur. [6 points]

(c) Use all of the information above to sketch a good graph of $y = f(x)$ in the space below. [5 points]

8. Find the exact area of the region in the $xy-$plane that is bounded by the curve $y^2 = x - 2$ and the lines $y = 1$, $y = -x$, and $y = -1$. (A complete solution requires a neat diagram and sufficient details.) [10 points]

(This page is intentionally left blank)