***** Sorry - Solutions will not be posted *****

FINAL EXAMINATION

MATA32 - Calculus for Management I

Examiners: R. Grinnell E. Moore
X. Jiang T. Pham

Date: December 11, 2007
Duration: 3 hours

Provide the following information:

Family Name (PRINT): _____

Given Name(s) (PRINT): _____

Student Number (PRINT): _____

Signature: _____

Read these instructions:

1. This examination has 15 numbered pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.
2. Put your solutions and/or rough work in the answer space provided. If you need extra space, use the back of a page or the blank page at the end of the exam.
3. You may use one standard hand-held calculator. All other electronic devices, extra paper, notes, and textbooks are forbidden at your workspace.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	8	9	**TOTAL**
45	11	9	20	10	8	11	12	13	11	150

The following formulas may be helpful:

$$S = P(1+r)^n \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$\sum_{k=1}^{n} c = cn \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Profit = Revenue - Cost

Part A: Multiple Choice Questions For each of the following, **print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 4.5 points and no answer/wrong answers earn 0 points. Justification is neither required nor rewarded, but a small workspace is provided for your calculations.

1. If y is assumed to be defined implicitly as a function of x by the equation $xy + ln(y) = 3$ then y' evaluated at $x = 3$ and $y = 1$ is equal to

   (a) -1 (b) $-\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{4}$

2. The exact value of $\int_0^2 6x\sqrt{1+2x^2}\,dx$ is

   (a) 156 (b) 26 (c) 13 (d) 27

3. For $x > 0$, let $g(x) = x^k$ where $k \in (0, 1)$ is a constant. We may conclude that

   (a) $g'(x)$ is increasing on $(0, \infty)$ (b) $g'(x)$ is decreasing on $(0, \infty)$

   (c) $g(x)$ is decreasing on $(0, \infty)$ (d) none of (a), (b), or (c) is true.

4. If an investment increases by 500 % over a 15 year period under a constant periodic interest rate of r % monthly, then the annual interest rate is approximately

   (a) 10.78 % (b) 11.32 % (c) 12.69 % (d) 12.00 %

5. If $f(x) = e^{-4x} + 4x$ then

   (a) $f^{(n)}(x) = (-1)^n 4^n e^{-4x}$ for all $n \geq 1$ (b) $f^{(n)}(x) = -(4^n e^{-4x})$ for all $n \geq 2$

   (c) $f^{(n)}(x) = (-4)^n e^{-4x}$ for all $n \geq 2$ (d) none of (a), (b), or (c) is true.

6. If a marginal cost function is $c'(q) = 6q^2 + 880$ dollars per unit, then the cost to increase production from 6 units to 9 units is

   (a) $\$\ 3,666$ (b) $\$\ 2,694$ (c) $\$\ 270$

   (d) impossible to find, as we do not know the exact production cost for 6 or 9 units.

7. If $y = (ex)^{\sqrt{x}}$ then $y'(1)$ equals

   (a) e (b) 1 (c) $1 + \frac{e}{2}$ (d) $\frac{3e}{2}$

8. If for all real x we assume $f(g(x)) = x$ and $f'(x) = 1 + [f(x)]^2$, then $g'(0)$ equals

   (a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

9. If $\int f(x)\,dx = \dfrac{x+1}{x+2} + 3$ then

(a) $f(x) = 1 - ln(x+2)$ (b) $f(x) = \dfrac{x^2+2x}{x^2+4x}$ (c) $f(x) = \dfrac{1}{(x+2)^2}$

(d) none of (a), (b), or (c) is true.

10. If f is a function such that $f''(x) < 0$ for all $x > 2$ and $f''(x) > 0$ for all $x < 2$, then

(a) f must be increasing on $(-\infty, 2)$

(b) f has a point of inflection at $x = 2$

(c) the line $x = 2$ could be a vertical asymptote of f

(d) f must have a relative maximum at $x = 2$

(Did you print the letters of the correct answers in the boxes on the first page ?)

Part B: Full-Solution Questions Write clear and neat solutions in the answer spaces provided. Show all of your work. Full points are awarded for solutions only if they are correct, complete, and sufficiently display relevant concepts from MATA32.

1. In all of this question assume the cost to produce and sell every 100 units of a product is \$ 30 and that the demand function is $p = \dfrac{100}{\sqrt{q}}$ dollars per 100 units, where $p, q > 0$

   (a) State the profit function, $F(q)$. [3 points]

   (b) Calculate the absolute maximum profit and the level of production at which the maximum profit occurs under the extra assumption that $q \in [3, 8]$. Round your final answers to 2 decimals. Sufficiently justify your solution. [8 points]

2. (a) Use appropriate mathematical notation and terminology to give a complete statement of the Fundamental Theorem of Calculus. [4 points]

(b) Give the mathematical formula for effective rate of interest. In at most a couple of sentences, explain in words the concept of effective rate of interest.

[5 points]

3. Find the following

(a) $\int \frac{(t^2+3t)^2}{t^3}\,dt$ [5 points]

(b) $\int_0^2 3x^5 e^{x^3} dx$ [9 points]

(c) $\int_1^e \frac{(ln(x))^{1/n}}{x}\, dx$ where n is a natural number. [6 points]

4. On the axis below, accurately sketch the graph of a function $y = f(x)$ having **all** of the following properties:

   (i) The domain is $\{x \in \mathbf{R} \mid x \neq 1\}$

   (ii) $f(0) = 0$ and $f(-\frac{1}{2}) = 1$

   (iii) $\lim_{x \to -\infty} f(x) = 2$ and $\lim_{x \to \infty} f(x) = 2$

   (iv) $f'(x) = \dfrac{-2x}{(x-1)^3}$

   (v) $f''(x) = \dfrac{2(2x+1)}{(x-1)^4}$

   A complete answer is one that displays relevant details/calculations as well as an accurate graph. [10 points]

5. Find the equation of the line having negative slope, that passes through the point $(2, -3)$, and is tangent to the curve $y = x^2 + x$. [8 points]

6. A rectangular storage container has a volume of 10 m^3 and no top. The length is twice the width, the base material costs $\$10/m^2$, and the side material costs $\$6/m^2$. Calculate the dimensions of the box (i.e. length, width, and height, to the nearest cm) that minimize the total cost of materials. Sufficiently justify your solution. [11 points]

7. (a) Calculate the maximum possible amount of compound interest than can be earned on any deposit over 10 years with 6.9 % APR. Express your final answer as a percentage rounded to 2 decimals. [6 points]

(b) Starting at age 20, you deposit \$ R into a savings account at the end of each quarter for the next 35 years. At the end of this time, no further deposits are made until your retire at age 65. If interest in the account is fixed at 5.2 % APR compounding quarterly, find the amount of the quarterly deposit, R, so that you will have exactly \$1 million in the account when you retire. Round your final answer to the nearest cent. [6 points]

8. In all of this question let $I = \int_{-2}^{2} f(x)dx$ where $f(x) = -2x + 1$.

(a) Evaluate I. [4 points]

(b) Evaluate I using the definition of the definite integral: $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$ [6 points]

(c) Find $\int_{-2}^{2} |f(x)|\, dx$ [3 points]

9. Find the area of the region in the xy-plane that is bounded by the curve $x = y^2$ and the line $2y + x = 3$. A complete answer includes a diagram and sufficient details/calculations.

[11 points]

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