University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION
MATA32S - Calculus for Management I

*****Solutions are not provided*****

Examiner: R. Grinnell

Date: April 29, 2013
Time: 2:00 pm
Duration: 3 hours

Lastname (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number: ______________________________

Signature: ______________________________

Instructions:

1. It is your responsibility to ensure that all 13 pages of this examination are included.
2. If you need extra space, use the back of a page or blank page 13. Clearly indicate the location of your continuing work.
3. Show all work and justify your answers. Full points are awarded for solutions that are correct, complete, and show appropriate concepts from MATA32S.
4. You may use one calculator that does not perform any: graphing, matrix operations, numerical/symbolic differentiation or integration. Cell/smart phones, i-Pods/Pads, other electronic transmission/receiving devices, extra paper, notes, textbooks, pencil/pen cases, food, drink boxes/bottles with labels, and backpacks are forbidden at or under your workspace.
5. You may write in pen, pencil, or other ink.

Print letters for the Multiple Choice Questions in these boxes

1	2	3	4	5	6	7	8	9	10

Do not write anything in these boxes

A	1	2	3	4	5	6	7	8	**TOTAL**
30	14	16	14	16	15	15	14	16	150

Some formulas:

$$S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right] \qquad S = Pe^{rt} \qquad NPV = \left(\sum PV\right) - P$$

Part A: Ten Multiple Choice Questions Clearly print the letter of the answer you think is most correct in the boxes on the first page. Right answers earns 3 points and no answer/wrong answers earn 0 points. Justification is not required or rewarded.

1. The equation of the tangent line to the curve $y = (18-2x)^{1/3}$ at the point where $x = 5$ is
   (A) $5x - 6y = 13$ (B) $x - 6y = -7$ (C) $y + 6x = 17$ (D) $x + 6y = 17$
   (E) none of (A) - (E)

2. What annual percentage rate of interest compounded quarterly is most closely equivalent to 4% APR compounding continuously ?

   (A) 4.02% (B) 4.08% (C) 4.11% (D) 4.039% (E) none of (A) - (D)

3. A cost function is given by $c(x) = 44x + \dfrac{16}{x} + 8$ where $x > 0$ is quantity. The marginal average cost when $x = 2$ is

   (A) 40 (B) 2 (C) -2 (D) -6 (E) -8 (F) a number not in (A) - (E)

4. If y is defined implicitly as a function of x by the equation $xy^2 + y = 4x$ then the value of $\dfrac{dy}{dx}$ when $x = 0$ is (A) 0 (B) 2 (C) 4 (D) -4

(E) a number not in (A) - (D) (F) unknown because y is not given

5. If $f(x) = \left(2x + 3\right)^x$ then $f'(0)$ equals

(A) 0 (B) $\dfrac{2}{3} + ln(3)$ (C) $ln(27)$ (D) $ln(3)$ (E) 1 (F) none of (A) - (E)

6. The value of $\int_0^1 \left(1 + x + x^2 - x^5\right) dx$ is (A) $\dfrac{49}{30}$ (B) 2 (C) $\dfrac{8}{3}$ (D) $\dfrac{5}{3}$

7. If $y' = 3x^2 + x$ and $y(2) = 4$ then $y(1)$ equals

(A) $\dfrac{9}{2}$ (B) $\dfrac{-15}{2}$ (C) -7 (D) $\dfrac{-5}{2}$ (E) none of (A) - (D)

8. The set of critical numbers of $h(x) = ln\left(x^2 - 4\right)$ is (A) $\{0\}$ (B) $\{0, -2\}$ (C) $\{0, 2\}$ (D) $\{0, -2, 2\}$ (E) empty because h does not have any critical numbers.

9. The area of the region lying beneath $y = 6x^2 + 6$ where $-3 \leq x \leq 1$ is

(A) 16 (B) 80 (C) 28 (D) 32 (E) 40 (F) none of (A) - (E)

10. Exactly how many of the following four statements are always true ?

(i) If h is continuous for all real x and $h(x) > 0$ for all $x > 0$, then $\int_a^b h(x)\,dx$ represents the area under the curve $y = h(x)$ where $a \leq x \leq b$.

(ii) The definite integral is a function obtained by taking the limit of a special kind of sum.

(iii) If F and G are antiderivatives of a continuous function f, then $F(x) = G(x)$.

(iv) $\int t^n \, dt = \dfrac{t^{n+1}}{n+1} + C$ for all constants n (C is the constant of integration).

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

<u>BE SURE YOU HAVE PUT YOUR LETTERS IN THE PAGE 1 BOXES</u>

Part B: Eight Full-Solution Questions Write clear, neat solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32S.

1. The parts of this question are independent of each other.

   (a) Find $f'(1)$ where $f(x) = \sqrt{(x+1)(x^2+1)(x^3+1)}$. Give your simplified final answer in terms of mathematical constants, not decimals. [7 points]

   (b) Evaluate $\int_1^e \frac{1+\left(ln(t)\right)^2}{t}\,dt$. [7 points]

2. The parts of this question are independent of each other.

(a) Find the value of the constants A and B such that the function $y = (Ax + B)e^{-x}$ satisfies the equation $y - y' = 4xe^{-x}$. [9 points]

(b) In all of this question let α be a positive constant, u be a differentiable function for all real values x, and let $w = (1 + \alpha)u - \alpha x$.

(i) Show that $\dfrac{dw}{dx} = \beta\left(\dfrac{du}{dx} - \dfrac{\alpha}{\beta}\right)$ where $\beta = 1 + \alpha$. [4 points]

(ii) If $\dfrac{du}{dx} < \dfrac{\alpha}{\beta}$ for all x, show that w is a decreasing function of x. [3 points]

3. The parts of this question are independent of each other.

(a) P owes Q \$5,000 now and agrees to pay Q the sum of \$1,000 at the end of each year for five years and a final payment at the end of the sixth year. Interest is 4% APR compounding annually. Find the amount of the final payment, rounded up to the nearest dollar. [7 points]

(b) In this part, assume all money is in units of \$1,000's. Suppose an initial investment of 37 in a small business guarantees the following cash flows:

Year	1	2	3	4
Cash Flow	8	13	15	F

Interest is 2% APR compounding annually. Cash flow payments are made at the end of each of the four years as in the table above. Find the smallest value of F above (rounded up to the nearest hundred dollars) so that the business investment is profitable. [7 points]

4. Find the integrals.

(a) $\displaystyle\int \frac{1}{x^3}\sqrt{\frac{8x^2+1}{4x^2}}\,dx$ (Hint: manipulate the integrand first.) [6 points]

(b) $\displaystyle\int_0^4 e^{\sqrt{x}}\,dx$ (Give your answer in terms of mathematical constants, not decimals.)

[10 points]

5. In all of this question let $f(x) = x^2 - 15$.

(a) State the Newton's method formula applied to $f(x)$. and simplify your answer in a way that minimizes the instances of x_n. [4 points]

(b) Find the positive integer a such that a root of f lies in the interval $(a, a+1)$. Sufficiently justify your work. [3 points]

(c) Let x_1 be the smaller of $|f(a)|$ and $|f(a+1)|$ and use part (a) to find x_2 and x_3. Round your final answer for x_3 to four decimals. What real number are these approximations to ?

[8 points]

6. In all of this question consider the region $\mathcal{R} = \{(x, y) \mid 0 \leq y \leq 2,\ -y^2 + 1 \ \leq x \ \leq y + 3\}$.

(a) Draw an accurate, labeled sketch of the region $\mathcal{R}$ and find its area using the method of horizontal strips. [11 points]

(b) State the sum of integrals that would give the area of $\mathcal{R}$ if it were calculated using vertical strips. Do not evaluate these integrals. [4 points]

7. In all of this question, let $h(x) = 4x^{3/5} - x^{8/5}$. [14 points]

(a) State the domain and x-intercepts of h.

(b) Find all critical values of h.

(c) State all intervals where h is increasing/decreasing and state all local extrema of h.

(d) Make an accurate, neat sketch of the function $y = h(x)$ that shows the features in parts (a) - (c). You need not find/use the second derivative.

8. In all of this question a car travels a constant speed of $x > 0$ kilometres per hour over a 300 kilometre trip. Assume the car's fuel consumption in litres per kilometre is given by the function $F(x) = \frac{1}{300}\left(\frac{1500}{x} + x\right)$. The driver of the car is paid \$25 per hour and gas for the car costs \$1.25 per litre.

   (a) State the total cost function $C(x)$, which is the sum of the driver's pay and the total fuel cost, for the 300 kilometre trip. Simplify $C(x)$. [7 points]

   (b) Calculate the constant speed that results in the smallest total cost for the trip. Round your final answer up to one decimal. State the smallest trip cost rounded up to the nearest dollar. Sufficiently justify your solution. [9 points]

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